GENERALIZED MINKOWSKI-FUNK TRANSFORMS AND SMALL DENOMINATORS ON THE SPHERE *

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Abstract

The Cauchy problem for the Euler-Poisson-Darboux equation on the unit sphere S^n gives rise to a family of fractional integrals $M^{\alpha}_{\cos\theta} f(x)$ which integrate f over the spherical cap of radius θ centered at the point $x \in S^n$. These fractional integrals are called the generalized Minkowski-Funk transforms because various transforms of integral geometry (including those of Minkowski and Funk) are particular cases of $M^{\alpha}_{\cos\theta}$. The cases $\alpha = 0, 1$ and $(1-n)/2$ correspond to the spherical section transform, the spherical cap transform and the solution of the Cauchy problem for the wave equation on S^n respectively. Inversion of $M^{\alpha}_{\cos\theta}$ with fixed θ leads to the problem of small denominators which was not studied before in the context of the non-commutative harmonic analysis on spheres of dimension > 1. The structure of the kernel $\ker M^{\alpha}_{\cos\theta}$ and the behavior of $M^{\alpha}_{\cos\theta}$ in Sobolev spaces are investigated depending on α and arithmetical properties of θ. The paper sheds new light to the classical Schneider-Berenstein-Zalcman results on injectivity of the Pompeiu transform by giving them the relevant number-theoretical meaning.

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